MODEL EXAMINATION 2012-13

MATHEMATICS

CLASS:XII

BLUE PRINT

S.No	Topics	VSA (1 mark each)	SA (4 marks each)	LA (6 marks each)	Total
1.(a) (b)	Relations and Functions Inverse Trigonometric Functions	1(1) 1(1)	4(1) 4(1)		10(4)
2.(a) (b)	Matrices Determinants	2(2) 1(1)	4(1)	6(1)	13(5)
3.(a) (b) (c) (d) (e)	Continuity and differentiability Applications of derivatives Integration Applications of integrals Differential equations	2(2)	8(2) 4(1) 4(1) 8(2)	6(1) 6(1) 6(1)	44(11)
4.(a) (b)	Vectors 3-Dimensional geometry	2(2) 1(1)	4(1) 4(1)	6(1)	17(6)
5.	Linear programming			6(1)	6(1)
6.	Probability		4(1)	6(1)	10(2)
	Total	10(10)	48(12)	42(7)	100(29)

Marking Scheme

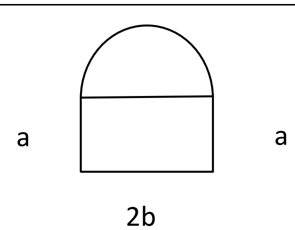
Q.NO	Value points/Ans	Marks
1	14	1
2	-7/17	1
3	K=3/2	1
4	α=π/3	1
5	A=$\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$ B=$\begin{pmatrix} 0 & 0 \\ 0 & 5 \end{pmatrix}$	1
6.	$\log \| x + \log \sin x \| + c$	1
7	1	1
8	5√2	1
9	a= -40	1
10	$\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$	1

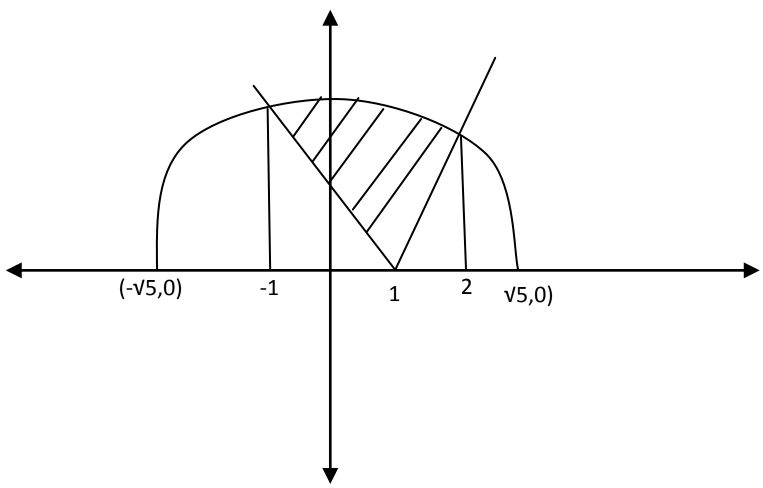
11	For $x \in N$, $x^2-4xx+3x^2=0$ $(x,x) \in R$ R is reflexive $(3,1) \in R$ as $3^2+4.3.1+3.1^2=0$ But $1^2-4.1.3+3.3^2 \neq 0$ So $(1,3) \notin R$ R is not symmetric $(9,3) \in R$ as $9^2-4.9.3+3.3^2=0$ $(3,1) \in R$ as $3^2+4.3.1+3.1^2=0$ But $(9,1) \notin R$ as $9^2-4.9.1+3.1^2 \neq 0$ R is not transitive	1 1 ½ 1 ½
12	$2[tan^{-1}\left(\frac{1}{5}\right)+sec^{-1}\left(\frac{5\sqrt{7}}{7}\right)=2tan^{-1}\left[\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}\times\frac{1}{8}}\right]+sec^{-1}\left(\frac{5\sqrt{7}}{7}\right)$ $=2tan\left(\frac{\frac{13}{40}}{1-\frac{1}{40}}\right)+tan^{-1}\left(\frac{1}{7}\right)$ $=2tan^{-1}\left(\frac{1}{3}\right)+tan^{-1}\left(\frac{1}{7}\right)$ $=tan^{-1}\left(\frac{\frac{2}{3}}{1-\frac{1}{9}}\right)+tan^{-1}\left(\frac{1}{7}\right)$ $=tan^{-1}\left(\frac{3}{4}\right)+tan^{-1}\left(\frac{1}{7}\right)$ $tan^{-1}1=\frac{\pi}{4}$ (OR) $Sin^{-1}(5/x)+sin^{-1}(12/x)=\pi/2$ $12/x=sin(\pi/2-sin^{-1}5/x)=cos(sin^{-1}5/x)$ Let $sin^{-1}x=t$ $5/x=sint$ $Cost=\sqrt{1-sin^2t}$ $=\sqrt{1-\frac{25}{x^2}}$ $t=cos^{-1}\sqrt{1-\frac{25}{x^2}}$ $12/x=\sqrt{1-\frac{25}{x^2}}$ $144/x^2=(x^2-25)/x^2$ $X=\pm 13$ But $x=-13$ does not satisfy the equation $X=13$	1 1 1 1 1 1/2 1/2 1/2 1/2 1/2 1/2

13	Applying $R_1 \to aR_1, R_2 \to bR_2, R_3 \to cR_3$ and multiply whole determinant by 1/abc Taking out common factors a,b,c from C_1, C_2, C_3 respectively Applying $R_1 \to R_1+R_2+R_3$ Taking out 2 from R_1 Applying $R_2 \to R_2-R_1, R_3 \to R_3-R_1$ Applying $R_1 \to R_1+R_2+R_3$ On expanding along Ist row $2\{-c^2(-b^2a^2)+b^2(c^2a^2)\}=4a^2b^2c^2$	1/2 ½ 1 1 ½ ½
14	f(x) is cont. at x=π/4 LHL=RHL π/4+a√2sin π/4=2x π/4cot π/4+b a-b= π/4-----------------------(1) f(x) is cont.at x= π/2 2x π/2cot π/2+b=a(-1)b a+2b=0------------------------(2) soving (1)&(2) getting a=3 π/2&b=-3 π/4	½ 1 ½ 1 1
15	Getting $x = \frac{cosy}{\cos(a+y)}$ $\frac{dx}{dy} = \frac{\cos(a+y)(-siny) - cosy(-\sin(a+y))}{cos^2(a+y)}$ $= \frac{\sin(a+y-y)}{cos^2(a+y)}$ $= \frac{sina}{cos^2(a+y)}$ $\frac{dy}{dx} = \frac{cos^2(a+y)}{sina}$ OR Getting dx/dt=a(1+cost) &dy/dt=-asint dy/dx+-asint/a(1+cost)=-tan(t/2) $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(-tan\frac{t}{2}\right)\frac{dt}{dx}$ $= -sec^2\left(\frac{t}{2}\right) \times \frac{1}{2} \times \frac{1}{a(1+cost)}$ $\frac{d^2y}{dx^2} = -\frac{2}{2a} = -1/a$	1 1 1 1 1 1 1 1
16	$f'(x)=e^{x(1-x)}+x\, e^{x(1-x)}(1-2x)$ $=-e^{x(1-x)}(x-1)(2x+1)$ F'(x)=0 gives x=1 or -1/2 In (-∞,-1/2) ,function is decreasing (-1/2,1),function is increasing (1, ∞),function is decreasing	1 1/2 1 ½ ½ ½

17	$Let\ I = \int \frac{x+1}{x(1+xe^x)^2}dx$ Put $1+xe^x=t$ implies $e^x(x+1)dx=dt$ $I=\int \frac{dt}{(t-1)t^2}$ Let $\frac{1}{(t-1)t^2} = \frac{A}{t-1} + \frac{B}{t} + \frac{c}{t^2}$ Getting A=1,B=-1,C=-1 $I=\int \left(\frac{1}{t-1} - \frac{1}{t} - \frac{1}{t^2}\right) dt$ $= log\|t-1\| + log\|t\| + \frac{1}{t} + c$ $= log\left\|\frac{xe^x}{1+xe^x}\right\| + \frac{1}{1+xe^x} + c$ OR $I=\int \sqrt{sin^2\frac{x}{4} + cos^2\frac{x}{4} + 2sin\frac{x}{4}cos\frac{x}{4}}dx$ $= \int \sqrt{(sin\frac{x}{4} + cos\frac{x}{4})^2}dx$ $= \int \left(sin\frac{x}{4} + cos\frac{x}{4}\right)dx = -4cos\frac{x}{4} + 4sin\frac{x}{4} + c$	1/2 1/2 1 1 1 2 1 1
18	Correct differentiation w.r.t. x Obtaining$\frac{dy}{dx} - \frac{y}{x} + be^{\frac{y}{x}} = 1$ Again differentiating w.r.t x $\frac{d^2y}{dx^2} - \frac{x\frac{dy}{dx} - y}{x^2} + be^{\frac{y}{x}}\left[\frac{x\frac{dy}{dx} - y}{x^2}\right] = 0$ $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$ OR Obtaining eqn. of circle as $(x+a)^2+(y-a)^2=a^2$ Diiferentiating w.r.t x $x + y\frac{dy}{dx} = a\left(\frac{dy}{dx} - 1\right)$ $a = \frac{x+yy'}{y'-1}$ $putting\ the\ value\ of\ a\ in\ (1)\left(x + \frac{x+y'}{y'-1}\right)^2 + \left(y - \frac{x+yy'}{y'-1}\right)^2 = \left(\frac{x+yy'}{y'-1}\right)^2$ Obtaining$(x+y)^2(y'^2+1)=(x+yy')^2$	1 1 1 1 1 1 ½ ½ 1

19	Getting dy/dx=y/x - $\sin^2(y/x)$ Put y=vx then dy/dx=v+x(dv/dx) Getting $v + x\frac{dv}{dx} = v - \sin^2 v$ $-\text{cosec}^2 v\,dv = dx/x$ Integrating cotv=logx+c Cot(y/x)-logx=c	½ 1 1 ½ ½ ½
20	R.H.S=$1 + (\vec{a}.\vec{b})^2 - 2\vec{a}.\vec{b} + \|\vec{a}\|^2 + \|\vec{b}\|^2 + 2\vec{a}.\vec{b} + \|\vec{a} \times \vec{b}\|^2 - (\vec{a} + \vec{b}).(\vec{a} \times \vec{b})$ Simplifying and put $\vec{a}.\vec{b} = \|\vec{a}\|\|\vec{b}\|\cos\theta$ and $\vec{a} \times \vec{b} = \|\vec{a}\|\|\vec{b}\|\sin\theta\hat{n}$ *simplifying in the form* $= 1 + \|\vec{a}\|^2\|\vec{b}\|^2 + \|\vec{a}\|^2 + \|\vec{b}\|^2 = (1 + \|\vec{a}\|^2)(1 + \|\vec{b}\|^2)$	1 2 1
21	Let $\frac{x-1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \tau$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu$ If they intersect, then for some τ and μ $3\tau - 1 = \mu + 2$,---------(1) $5\tau - 3 = 3\mu + 4$,-----------(2) $7\tau - 5 = 5\mu + 6$-------------(3) Solving (1)&(2) and getting $\tau = 1/2, \mu = -3/2$ Which satisfies (3) Hence lines intersect Point of contact(1/2,-1/2,-3/2)	½ ½ ½ ½ 1 ½ ½
22	Getting random variable as 3,4,5,6,7,8,9 X: 3, 4, 5, 6, 7, 8, 9 P(X): 1/10, 1/10, 1/5, 1/5, 1/5, 1/10, 1/10	½ For each Correct Probability ½
23	$\|A\| \neq 0$ A is invertible Finding $A^{-1}=1/10\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ Writing $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$ $A^T X = B$ Where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$	½ 2 ½ ½ ½

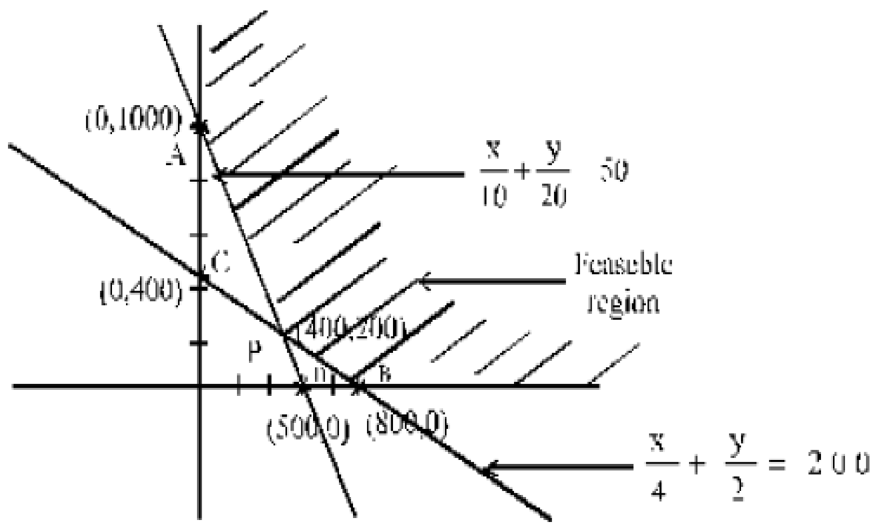
	$\lvert A\rvert = \lvert A^T\rvert \neq 0$	
	$(A^T)^{-1} = (A^{-1})^T = 1/10\begin{bmatrix}4 & -5 & 1\\ 2 & 0 & -2\\ 2 & 5 & 3\end{bmatrix}$	½ ½
	X=$(A^T)^{-1}$B	
	X=$\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}\frac{9}{5}\\ \frac{2}{5}\\ \frac{7}{5}\end{bmatrix}$	1
	$x = \frac{9}{5}, \quad y = \frac{2}{5}, \quad z = \frac{7}{5}$	
	OR	
	A=IA	
	$\begin{bmatrix}1 & 3 & -2\\ -3 & 0 & -5\\ 2 & 5 & 0\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A$	1
	$R2 \to R2 + 3\,R1, R3 \to R3 - 2\,R1$ *and getting correct equality*	1
	$R1 \to R1 + 3R3$ *and getting correct equality*	½
	$R2 \to R2 + 8R3$*and getting correct equality*	½
	$R3 \to R3 + R2$ *and getting correct equality*	½
	$R3 \to \frac{1}{25}R3$ *and getting correct equality*	½
	$R1 \to R1 - 10R3, \quad R2 \to R2 - 21R3$ *and getting correct equality*	1
	$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & \frac{-10}{25} & \frac{-15}{25}\\ \frac{-10}{25} & \frac{4}{25} & \frac{11}{25}\\ \frac{-15}{25} & \frac{1}{25} & \frac{9}{25}\end{bmatrix}A$	
	$A^{-1} = \frac{1}{25}\begin{bmatrix}25 & -10 & -15\\ -10 & 4 & 11\\ -15 & 1 & 9\end{bmatrix}$	1
24	 	1
	Perimeter P=2a+4b+πb--------------------(1)	1/2
	Let the transmission rate of coloured glass be L and Q be total transmitted light	
	Q=2ab(3L)+1/2πb^2(L)	1

	Q=L/2(6Pb-24b^2-5πb2) using (1) dQ/db=L/2(6P-48b-10πb) applying dQ/db=0 implies $b=\frac{6P}{48+10\pi}$ $\frac{d^2Q}{db^2}=\frac{L}{2}(-48+10\pi)<0$ so Q is maximum now (48+10π)b=6P=6(2a+4b+πb) 2b:a=6:(6+π)	1/2 1/2 1 1/2 1
25	 Solving the equations and obtaining x=2,-1 Required area=$\int_{-1}^{2}\sqrt{5-x^2}\,dx-\int_{-1}^{1}(-x+1)dx-\int_{1}^{2}(x-1)dx$ Evaluation and simplification Obtain area as 5π/4-1/2	2 1 1 1 1
26	I=$\int_0^{\pi}\frac{x\sin 2x\sin\left(\frac{\pi}{2}\cos x\right)}{2x-\pi}dx--------(1)$ $I=\int_o^{\pi}\frac{(\pi-x)\sin 2(\pi-x)\sin\left[\frac{\pi}{2}\cos(\pi-x)\right]}{2(\pi-x)-\pi}dx$ $I=\int_o^{\pi}\frac{(x-\pi)\sin 2x\sin\left[\frac{\pi}{2}\cos x\right]}{2x-\pi}dx------(2)$	1 ½ 1

	(1)+(2)	
	2I=$\int_0^{\pi} sin2x \sin\left(\frac{\pi}{2} cosx\right) dx$	1
	$2I = \int_0^{\pi} 2sinx \cos x \sin\left(\frac{\pi}{2} cosx\right) dx$	1
	Put π/2cosx = t so sinx dx = -2/π dx	½
	I = $\frac{-2}{\pi} \int_{\frac{\pi}{2}}^{-\frac{\pi}{2}} \frac{2t}{\pi} sint\ dt$	1
	Integrating and simplifying I=$\frac{8}{\pi^2}$	
	OR	
	$\|xsin\pi x\| = \begin{cases} xsin\pi x, -1 < x < 1 \\ -xsin\pi x, \ 1 < x < \frac{3}{2} \end{cases}$	2
	$I = \int_{-1}^{1} (xsin\pi x) dx + \int_{1}^{\frac{3}{2}} -xsin\pi x\ dx$	1
	$= 2\left[\frac{xcos\pi x}{\pi}\right]_0^1 - \int_0^1 \frac{-cos\pi x}{\pi} dx - \left[\frac{-xcos\pi x}{\pi}\right]_1^{\frac{3}{2}} + \int_1^{\frac{3}{2}} \frac{-cos\pi x}{\pi}\ dx$	2
	$=\frac{3}{\pi} + \frac{1}{\pi^2} = \frac{3\pi+1}{\pi^2}$	1
27	Let A(1,1,1) P(-3,1,5) pt on the line	1
	$\overrightarrow{AP} = -4\hat{\imath} + 4\hat{k}$	
	$\therefore$ *vector perpendicular to plane is*	$1^{1}/_{2}$
	$(-4\hat{\imath} + 4\hat{k}) \times (3\hat{\imath} - \hat{\jmath} + 5\hat{k}) = (\hat{\imath} - 2\hat{\jmath} + \hat{k})$	
	Eqn of plane	
	$\vec{r}.(\hat{\imath} - 2\hat{\jmath} + \hat{k}) = (\hat{\imath} + \hat{\jmath} + \hat{k}).(\hat{\imath} - 2\hat{\jmath} + \hat{k}) = 0$	$1^{1}/_{2}$
	$x + y + z = 0$	
	$now(\hat{\imath} - 2\hat{\jmath} - 5\hat{k}).(\hat{\imath} - 2\hat{\jmath} + \hat{k}) = 0$	1
	$\therefore \vec{r} = (-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}) + \lambda(\hat{\imath} - 2\hat{\jmath} + \hat{k})$ *is parallel to the plane*	
	Alsothe poinnt $(-1,2,5)$ *satisfies the plane*	1
	hence the plane contains the line	

28 Suppose xgms ofwheat and y gms of rice are mixed in the daily diet

Constraints 0.1x+0.05y≥50
0.25x+0.5y≥200
x/4+y/2≥200, x≥0,y≥0 2
Objective function
Minimize Z=$\frac{4x}{1000}+\frac{6y}{1000}$ 1/2



2

1

Feasible region is unbdd and has vertices A(0,1000) B(800,0) P(400,200)

Point		Z
A	(0,1000)	6
B	(800,0)	3.2
P	(400,200)	2.8

Min. Z=2.8

½

Wheat 400gms rice 200gms

29	E_1:Red ball is transferred from A to B E_2:Black ball is tranferred from A to B E:Red ball is drawn from B $P(E_1)=3/7$ $P(E_2)=4/7$ $P(E/E_1)=5/10=1/2$ $P(E/E_2)=4/10=2/5$ $P(E_2/E)=\frac{P\left(E/E_2\right)P(E_2)}{P\left(E/E_1\right)P(E_1)+P\left(E/E_2\right)P(E_2)}$ =16/31	1 3 1 1