

MODEL EXAMINATION 2012-13

MATHEMATICS

CLASS:XII

BLUE PRINT

S.No	Topics	VSA (1 mark each)	SA (4 marks each)	LA (6 marks each)	Total
1.(a)	Relations and Functions	1(1)	4(1)		10(4)
(b)	Inverse Trigonometric Functions	1(1)	4(1)		
2.(a)	Matrices	2(2)		6(1)	13(5)
(b)	Determinants	1(1)	4(1)		
3.(a)	Continuity and differentiability		8(2)		44(11)
(b)	Applications of derivatives		4(1)	6(1)	
(c)	Integration	2(2)	4(1)	6(1)	
(d)	Applications of integrals			6(1)	
(e)	Differential equations		8(2)		
4.(a)	Vectors	2(2)	4(1)		17(6)
(b)	3-Dimensional geometry	1(1)	4(1)	6(1)	
5.	Linear programming			6(1)	6(1)
6.	Probability		4(1)	6(1)	10(2)
	Total	10(10)	48(12)	42(7)	100(29)

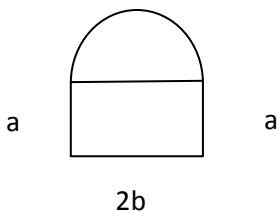
Marking Scheme

Q.NO	Value points/Ans	Marks
1	14	1
2	$-7/17$	1
3	$K=3/2$	1
4	$\alpha=\pi/3$	1
5	$A=\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} B=\begin{pmatrix} 0 & 0 \\ 0 & 5 \end{pmatrix}$	1
6.	$\log x + \log \sin x + c$	1
7	1	1
8	$5\sqrt{2}$	1
9	$a= -40$	1
10	$\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$	1

13	<p>Applying $R_1 \rightarrow aR_1, R_2 \rightarrow bR_2, R_3 \rightarrow cR_3$ and multiply whole determinant by $1/abc$</p> <p>Taking out common factors a, b, c from C_1, C_2, C_3 respectively</p> <p>Applying $R_1 \rightarrow R_1 + R_2 + R_3$</p> <p>Taking out 2 from R_1</p> <p>Applying $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$</p> <p>Applying $R_1 \rightarrow R_1 + R_2 + R_3$</p> <p>On expanding along 1st row $2\{-c^2(-b^2a^2) + b^2(c^2a^2)\} = 4a^2b^2c^2$</p>	<p>1/2</p> <p>1/2</p> <p>1</p> <p>1</p> <p>1/2</p> <p>1/2</p>
14	<p>$f(x)$ is cont. at $x = \pi/4$</p> <p>LHL=RHL</p> <p>$\pi/4 + a\sqrt{2}\sin \pi/4 = 2x \pi/4 \cot \pi/4 + b$</p> <p>$a - b = \pi/4$------(1)</p> <p>$f(x)$ is cont. at $x = \pi/2$</p> <p>$2x \pi/2 \cot \pi/2 + b = a(-1)b$</p> <p>$a + 2b = 0$------(2)</p> <p>solving (1)&(2) getting $a = 3\pi/2$ & $b = -3\pi/4$</p>	<p>1/2</p> <p>1</p> <p>1/2</p> <p>1</p> <p>1</p>
15	<p>Getting $x = \frac{\cos y}{\cos(a+y)}$</p> $\frac{dx}{dy} = \frac{\cos(a+y)(-\sin y) - \cos y(-\sin(a+y))}{\cos^2(a+y)}$ $= \frac{\sin(a+y-y)}{\cos^2(a+y)}$ $= \frac{\sin a}{\cos^2(a+y)}$ $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$ <p>OR</p> <p>Getting $dx/dt = a(1 + \cos t)$ & $dy/dt = -a \sin t$</p> <p>$dy/dx = -a \sin t / a(1 + \cos t) = -\tan(t/2)$</p> $\frac{d^2y}{dx^2} = \frac{d}{dt} \left(-\tan \frac{t}{2} \right) \frac{dt}{dx}$ $= -\sec^2 \left(\frac{t}{2} \right) \times \frac{1}{2} \times \frac{1}{a(1 + \cos t)}$ $\frac{d^2y}{dx^2} = -\frac{2}{2a} = -1/a$	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
16	<p>$f'(x) = e^{x(1-x)} + x e^{x(1-x)}(1-2x)$</p> <p>$= e^{x(1-x)}(x-1)(2x+1)$</p> <p>$f'(x) = 0$ gives $x = 1$ or $-1/2$</p> <p>In $(-\infty, -1/2)$, function is decreasing</p> <p>$(-1/2, 1)$, function is increasing</p> <p>$(1, \infty)$, function is decreasing</p>	<p>1</p> <p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>

17	<p>Let $I = \int \frac{x+1}{x(1+xe^x)^2} dx$</p> <p>Put $1+xe^x=t$ implies $e^x(x+1)dx=dt$</p> $I = \int \frac{dt}{(t-1)t^2}$ <p>Let $\frac{1}{(t-1)t^2} = \frac{A}{t-1} + \frac{B}{t} + \frac{C}{t^2}$</p> <p>Getting $A=1, B=-1, C=-1$</p> $I = \int \left(\frac{1}{t-1} - \frac{1}{t} - \frac{1}{t^2} \right) dt$ $= \log t-1 + \log t + \frac{1}{t} + c$ $= \log \left \frac{xe^x}{1+xe^x} \right + \frac{1}{1+xe^x} + c$ <p>OR</p> $I = \int \sqrt{\sin^2 \frac{x}{4} + \cos^2 \frac{x}{4} + 2\sin \frac{x}{4} \cos \frac{x}{4}} dx$ $= \int \sqrt{\left(\sin \frac{x}{4} + \cos \frac{x}{4} \right)^2} dx$ $= \int \left(\sin \frac{x}{4} + \cos \frac{x}{4} \right) dx = -4\cos \frac{x}{4} + 4\sin \frac{x}{4} + c$	<p>1/2</p> <p>1/2</p> <p>1</p> <p>1</p> <p>1</p> <p>2</p> <p>1</p> <p>1</p>
18	<p>Correct differentiation w.r.t. x</p> <p>Obtaining $\frac{dy}{dx} - \frac{y}{x} + be^{\frac{y}{x}} = 1$</p> <p>Again differentiating w.r.t x</p> $\frac{d^2y}{dx^2} - \frac{x \frac{dy}{dx} - y}{x^2} + be^{\frac{y}{x}} \left[\frac{x \frac{dy}{dx} - y}{x^2} \right] = 0$ $x^3 \frac{d^2y}{dx^2} = \left(x \frac{dy}{dx} - y \right)^2$ <p>OR</p> <p>Obtaining eqn. of circle as $(x+a)^2 + (y-a)^2 = a^2$</p> <p>Diiferentiating w.r.t x</p> $x + y \frac{dy}{dx} = a \left(\frac{dy}{dx} - 1 \right)$ $a = \frac{x + yy'}{y' - 1}$ <p>putting the value of a in (1) $\left(x + \frac{x + y'}{y' - 1} \right)^2 + \left(y - \frac{x + yy'}{y' - 1} \right)^2 = \left(\frac{x + yy'}{y' - 1} \right)^2$</p> <p>Obtaining $(x+y)^2(y'^2+1) = (x+yy')^2$</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1</p>

19	<p>Getting $dy/dx=y/x - \sin^2(y/x)$</p> <p>Put $y=vx$ then $dy/dx=v+x(dv/dx)$</p> <p>Getting $v + x \frac{dv}{dx} = v - \sin^2 v$</p> <p>$-\operatorname{cosec}^2 v dv = dx/x$</p> <p>Integrating $\cot v = \log x + c$</p> <p>$\cot(y/x) - \log x = c$</p>	<p>$\frac{1}{2}$</p> <p>1</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>																
20	<p>R.H.S = $1 + (\vec{a} \cdot \vec{b})^2 - 2\vec{a} \cdot \vec{b} + \vec{a} ^2 + \vec{b} ^2 + 2\vec{a} \cdot \vec{b} + \vec{a} \times \vec{b} ^2 - (\vec{a} + \vec{b}) \cdot (\vec{a} \times \vec{b})$</p> <p>Simplifying and put $\vec{a} \cdot \vec{b} = \vec{a} \vec{b} \cos\theta$ and $\vec{a} \times \vec{b} = \vec{a} \vec{b} \sin\theta\hat{n}$</p> <p><i>simplifying in the form</i></p> <p>$= 1 + \vec{a} ^2 \vec{b} ^2 + \vec{a} ^2 + \vec{b} ^2 = (1 + \vec{a} ^2)(1 + \vec{b} ^2)$</p>	<p>1</p> <p>2</p> <p>1</p>																
21	<p>Let $\frac{x-1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \tau$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu$</p> <p>If they intersect, then for some τ and μ</p> <p>$3\tau - 1 = \mu + 2, \dots \dots \dots (1)$</p> <p>$5\tau - 3 = 3\mu + 4, \dots \dots \dots (2)$</p> <p>$7\tau - 5 = 5\mu + 6, \dots \dots \dots (3)$</p> <p>Solving (1)&(2) and getting $\tau = 1/2, \mu = -3/2$</p> <p>Which satisfies (3)</p> <p>Hence lines intersect</p> <p>Point of contact $(1/2, -1/2, -3/2)$</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>																
22	<p>Getting random variable as 3,4,5,6,7,8,9</p> <table border="1" data-bbox="282 1360 1243 1436"> <tbody> <tr> <td>X</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> <td>9</td> </tr> <tr> <td>P(X)</td> <td>1/10</td> <td>1/10</td> <td>1/5</td> <td>1/5</td> <td>1/5</td> <td>1/10</td> <td>1/10</td> </tr> </tbody> </table>	X	3	4	5	6	7	8	9	P(X)	1/10	1/10	1/5	1/5	1/5	1/10	1/10	<p>$\frac{1}{2}$</p> <p>For each Correct Probability</p> <p>$\frac{1}{2}$</p>
X	3	4	5	6	7	8	9											
P(X)	1/10	1/10	1/5	1/5	1/5	1/10	1/10											
23	<p>$A \neq 0$ A is invertible</p> <p>Finding $A^{-1} = 1/10 \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$</p> <p>Writing $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$</p> <p>$A^T X = B$ Where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$</p>	<p>$\frac{1}{2}$</p> <p>2</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>																

	<p>$A = A^T \neq 0$</p> $(A^T)^{-1} = (A^{-1})^T = 1/10 \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$ <p>$X = (A^T)^{-1}B$</p> $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \\ 7 \\ 5 \\ 7 \\ 5 \end{bmatrix}$ <p>$x = \frac{9}{5}, \quad y = \frac{2}{5}, \quad z = \frac{7}{5}$</p> <p>OR</p> <p>$A=IA$</p> $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ <p>$R2 \rightarrow R2 + 3R1, R3 \rightarrow R3 - 2R1$ and getting correct equality</p> <p>$R1 \rightarrow R1 + 3R3$ and getting correct equality</p> <p>$R2 \rightarrow R2 + 8R3$ and getting correct equality</p> <p>$R3 \rightarrow R3 + R2$ and getting correct equality</p> <p>$R3 \rightarrow \frac{1}{25}R3$ and getting correct equality</p> <p>$R1 \rightarrow R1 - 10R3, \quad R2 \rightarrow R2 - 21R3$ and getting correct equality</p> $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{-10}{25} & \frac{-15}{25} \\ \frac{-10}{25} & \frac{4}{25} & \frac{11}{25} \\ \frac{-15}{25} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$ $A^{-1} = \frac{1}{25} \begin{bmatrix} 25 & -10 & -15 \\ -10 & 4 & 11 \\ -15 & 1 & 9 \end{bmatrix}$	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p>
24	 <p>Perimeter $P = 2a + 4b + \pi b$-----(1)</p> <p>Let the transmission rate of coloured glass be L and Q be total transmitted light</p> $Q = 2ab(3L) + \frac{1}{2}\pi b^2(L)$	<p>1</p> <p>$\frac{1}{2}$</p> <p>1</p>

	$Q=L/2(6Pb-24b^2-5\pi b^2)$ using (1) $dQ/db=L/2(6P-48b-10\pi b)$ applying $dQ/db=0$ implies $b = \frac{6P}{48+10\pi}$ $\frac{d^2Q}{db^2} = \frac{L}{2}(-48 + 10\pi) < 0$ so Q is maximum now $(48+10\pi)b=6P=6(2a+4b+\pi b)$ $2b:a=6:(6+\pi)$	1/2 1/2 1 1/2 1
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25	<p>Solving the equations and obtaining $x=2,-1$</p> <p>Required area=$\int_{-1}^2 \sqrt{5-x^2} dx - \int_{-1}^1 (-x+1)dx - \int_1^2 (x-1)dx$</p> <p>Evaluation and simplification Obtain area as $5\pi/4-1/2$</p>	2 1 1 1 1
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26	$I = \int_0^\pi \frac{x \sin 2x \sin\left(\frac{\pi}{2} \cos x\right)}{2x-\pi} dx$ ----- (1) $I = \int_0^\pi \frac{(\pi-x) \sin 2(\pi-x) \sin\left[\frac{\pi}{2} \cos(\pi-x)\right]}{2(\pi-x)-\pi} dx$ $I = \int_0^\pi \frac{(x-\pi) \sin 2x \sin\left[\frac{\pi}{2} \cos x\right]}{2x-\pi} dx$ ----- (2)	1 1/2 1
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	<p>(1)+(2)</p> $2I = \int_0^\pi \sin 2x \sin\left(\frac{\pi}{2} \cos x\right) dx$ $2I = \int_0^\pi 2 \sin x \cos x \sin\left(\frac{\pi}{2} \cos x\right) dx$ <p>Put $\pi/2 \cos x = t$ so $\sin x dx = -2/\pi dx$</p> $I = \frac{-2}{\pi} \int_{\frac{\pi}{2}}^{-\frac{\pi}{2}} \frac{2t}{\pi} \sin t dt$ <p>Integrating and simplifying</p> $I = \frac{8}{\pi^2}$ <p>OR</p> $ x \sin \pi x = \begin{cases} x \sin \pi x, & -1 < x < 1 \\ -x \sin \pi x, & 1 < x < \frac{3}{2} \end{cases}$ $I = \int_{-1}^1 (x \sin \pi x) dx + \int_1^{\frac{3}{2}} -x \sin \pi x dx$ $= 2 \left[\frac{x \cos \pi x}{\pi} \right]_0^1 - \int_0^1 \frac{-\cos \pi x}{\pi} dx - \left[\frac{-x \cos \pi x}{\pi} \right]_1^{\frac{3}{2}} + \int_1^{\frac{3}{2}} \frac{-\cos \pi x}{\pi} dx$ $= \frac{3}{\pi} + \frac{1}{\pi^2} = \frac{3\pi+1}{\pi^2}$	<p>1</p> <p>1</p> <p>½</p> <p>1</p> <p>2</p> <p>1</p> <p>2</p> <p>1</p>
27	<p>Let A(1,1,1) P(-3,1,5) pt on the line</p> $\vec{AP} = -4\hat{i} + 4\hat{k}$ <p>∴ vector perpendicular to plane is</p> $(-4\hat{i} + 4\hat{k}) \times (3\hat{i} - \hat{j} + 5\hat{k}) = (\hat{i} - 2\hat{j} + \hat{k})$ <p>Eqn of plane</p> $\vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = (\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0$ $x + y + z = 0$ <p>now $(\hat{i} - 2\hat{j} - 5\hat{k}) \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0$</p> <p>∴ $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \lambda(\hat{i} - 2\hat{j} + \hat{k})$ is parallel to the plane</p> <p>Also the point (-1,2,5) satisfies the plane</p> <p>hence the plane contains the line</p>	<p>1</p> <p>1^{1/2}</p> <p>1^{1/2}</p> <p>1</p> <p>1</p>

28

Suppose x gms of wheat and y gms of rice are mixed in the daily diet

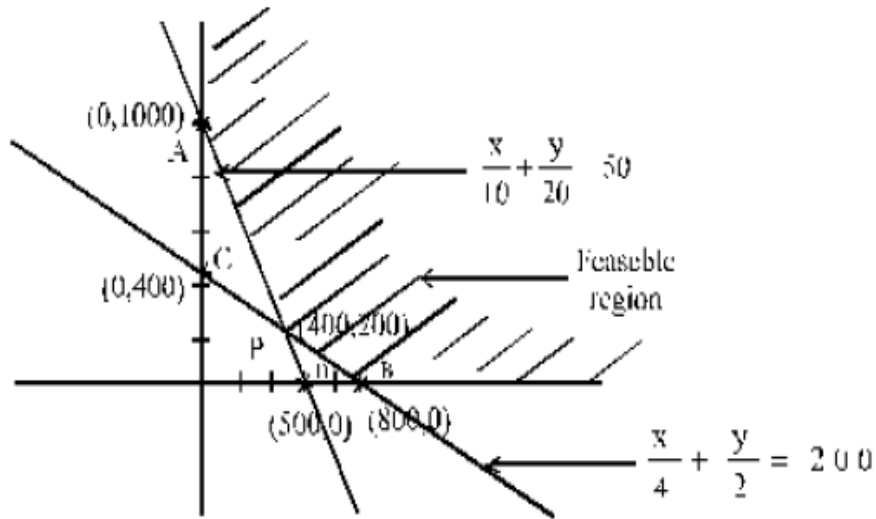
Constraints $0.1x + 0.05y \geq 50$

$0.25x + 0.5y \geq 200$

$x/4 + y/2 \geq 200, x \geq 0, y \geq 0$

Objective function

Minimize $Z = \frac{4x}{1000} + \frac{6y}{1000}$



Feasible region is unbdd and has vertices A(0,1000) B(800,0) P(400,200)

Point		Z
A	(0,1000)	6
B	(800,0)	3.2
P	(400,200)	2.8

Min. $Z=2.8$

Wheat 400gms rice 200gms

2

1/2

2

1

1/2

29	<p> E_1:Red ball is transferred from A to B E_2:Black ball is transferred from A to B E:Red ball is drawn from B $P(E_1)=3/7$ $P(E_2)=4/7$ $P(E/E_1)=5/10=1/2$ $P(E/E_2)=4/10=2/5$ </p> $P(E_2/E) = \frac{P(E/E_2)P(E_2)}{P(E/E_1)P(E_1) + P(E/E_2)P(E_2)}$ <p>=16/31</p>	<p>1</p> <p>3</p> <p>1</p> <p>1</p>
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